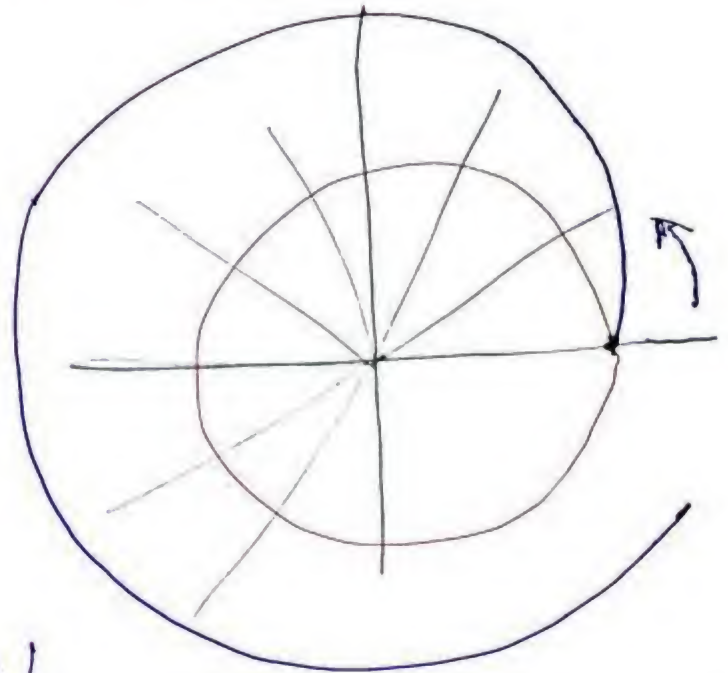
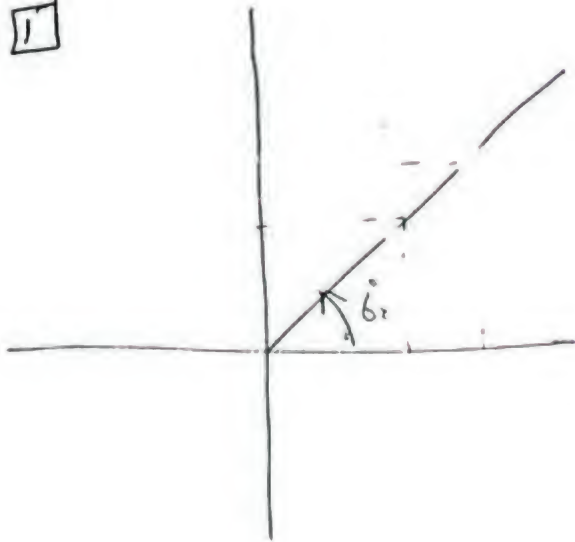


Lec 4 Digital Control

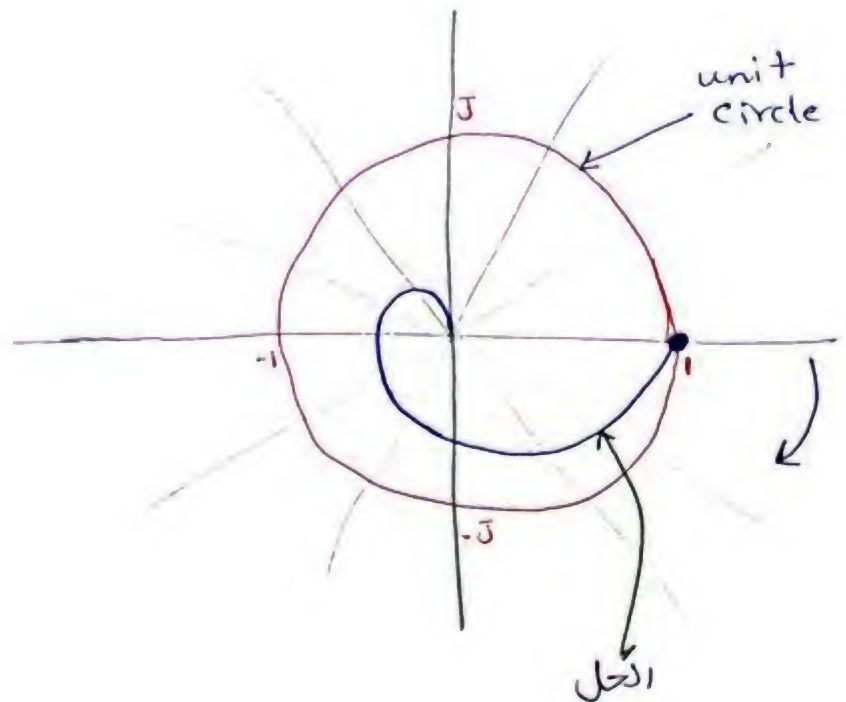
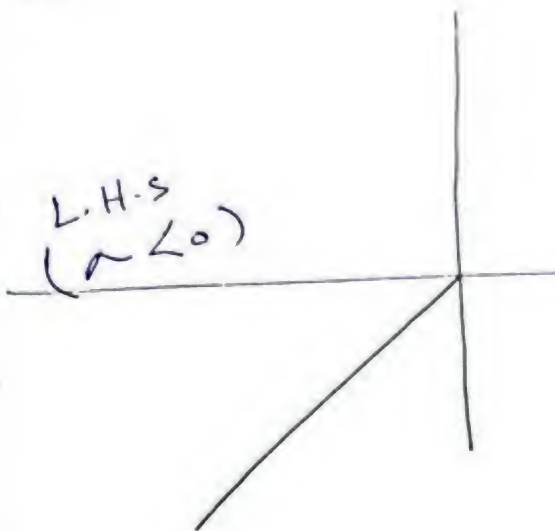
* map the following lines to Z-plane

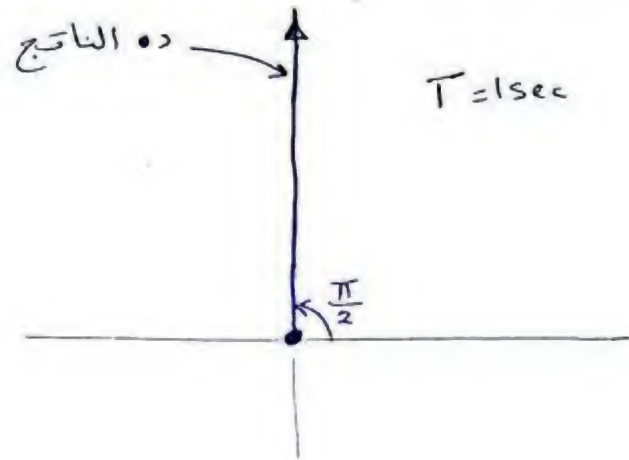
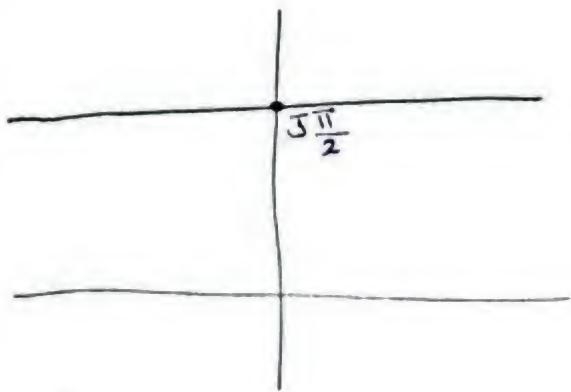
[1]



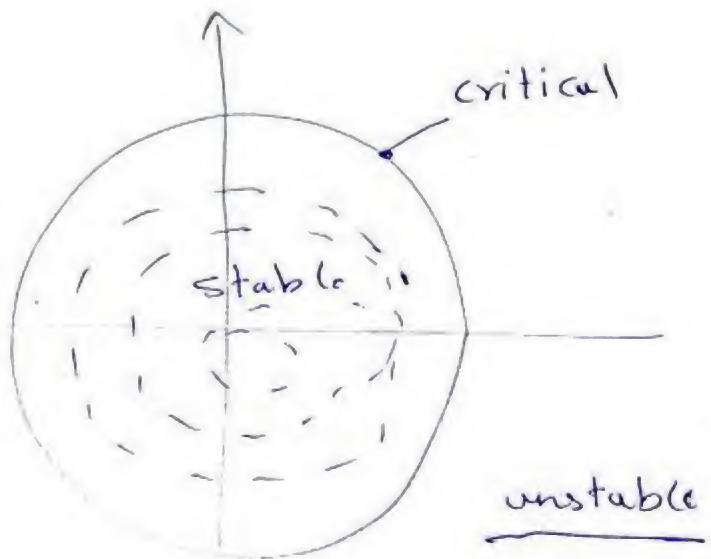
$$s = \sigma + j\omega \Rightarrow Z = e^{Ts} = |L| \angle \theta$$

[2]





$r = |z| =$
 $r = 1$ (critical)
 $r < 1$ (stable Pole)
 $r > 1$ (unstable Pole)



check system stability

I] Poles location s

Ex:1 ch. equation

$$z^2 - z + 0.5 = 0$$

$$\text{Poles} \Rightarrow z_{1,2} = 0.5 \pm j0.5$$

$$|z_{1,2}| = \sqrt{(0.5)^2 + (0.5)^2} = 0.707 < 1 \quad (\text{stable})$$

Ex2 ch. equation

$$(z - 0.5)(z + 0.7)(z - 1) = 0$$

Poles $z = 0.5, -0.7, 1$
1, 2, 3

$$|z| = 0.5, 0.7, 1$$

\uparrow \uparrow \uparrow
 < 1 < 1 $= 1$

\Rightarrow critically stable

← طالا مفيش حاجة خارج دائرة الوحدة.

Ex:3

$$(z - 0.5)(z + 0.1)(z - 1)(z + 1.5) = 0$$

~~hence~~ $z = 0.5, -0.1, 1, -1.5$

$$|z| = 0.5, 0.1, 1, 1.5$$

\uparrow \uparrow \uparrow \uparrow
 < 1 < 1 $= 1$ > 1
 \downarrow \uparrow
 (stable poles) (critical) (unstable)

\Rightarrow system unstable.

[2] using bilinear transformation:-

$$z = e^{\frac{T}{2}s} = e^{\frac{T}{2}s} \cdot e^{\frac{-T}{2}s} = \frac{e^{\frac{T}{2}s}}{e^{\frac{-T}{2}s}}$$

→ Taylor Expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

For small values of x

$$e^x \approx 1 + x$$

$$Z = \frac{e^{\frac{T}{2}s}}{e^{-\frac{T}{2}s}} = \frac{1 + \frac{T}{2}s}{1 - \frac{T}{2}s}$$

assume $\frac{T}{2}s = \gamma$

$$Z = \frac{1 + \gamma}{1 - \gamma}$$

→ (mapping) $J(s) \leftrightarrow Z(z)$
(γ -domain) (z -domain)

→ stability can be checked using Routh method.

Ex ch. equation

$$Z^3 + 3.3 Z^2 + 3 Z + 0.8 = 0$$

→ check stability using bilinear Transformation

$$Z = \frac{1 + \gamma}{1 - \gamma}$$

$$\left(\frac{1+r}{1-r}\right)^3 + 3 \cdot 3 \left(\frac{1+r}{1-r}\right)^2 + 3 \left(\frac{1+r}{1-r}\right) + 0.8 = 0$$

بالتقريب 3 $(1-r)^3$

$$(1+r)^3 + 3 \cdot 3 (1+r)^2 (1-r) + 3 (1+r) (1-r)^2 +$$

$$0.8 (1-r)^3 = 0$$

$$(1+r)(1+2r+r^2) + 3 \cdot 3 (1-r)(1+2r+r^2) +$$

$$3(1+r)(1-2r+r^2) + 0.8(1-r)(1-2r+r^2) = 0$$

$$\boxed{r^3 + 9r^2 - 9r - 81 = 0} \quad \left(\text{ch-equation in } r\text{-domain} \right)$$

	r^3	1	
	r^2	9	$\swarrow -9$
\rightarrow	r^1	0	$\searrow -81$
$A(r)$	r^0	-81	

$$A(r) = 9r^2 - 81$$

$$\frac{dA(r)}{dr} = 18r$$

(system unstable)

مع الإشارة لم تغيرت في آخر خطوة +ve و -ve (-ve)

Ex2 $Z^3 - 0.2Z^2 - 0.25Z + 0.05 = 0$

$$Z = \frac{1+r}{1-r}$$

ch. eqn.

$$\left(\frac{1+r}{1-r}\right)^3 - 0.2\left(\frac{1+r}{1-r}\right)^2 - 0.25\left(\frac{1+r}{1-r}\right) + 0.05 = 0$$

$$(1+r)^3 - 0.2(1+r)^2(1-r) - 0.25(1+r)(1-r)^2 + 0.05(1-r)^3 = 0$$

$0.9r^3 + 3.6r^2 + 2.9r + 0.6 = 0$

→ ch. equation in r -domain

r^3	0.9	2.9
r^2	3.6	0.6
r^1	2.75	
r^0	0.6	

* There are no sign changes in first column:
system stable.

[K]

3) using Jury Test

مع (Jury) على معنوفة قدر يحسب بها ال (stability)
← شغالي ال (Z-domain).

assume That the ch. eqn. $\Rightarrow F(z)$

$$F(z) = 1 + \overline{GH(z)} = 0$$

→ General Form

$$F(z) = \underline{a_n} z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \dots + \underline{a_0}$$

↳ $F(z) = 1 + \overline{GH(z)}$

→ The system to be stable

1) $F(1) > 0$

2) $(-1)^n F(-1) > 0 \rightarrow n \rightarrow \text{system order.}$

3) $|a_0| < |a_n|$

← الثلاثة فقط دي تستخدم في حالة (2nd order system)

لكن لو (system order) أكبر من 2 ← نستخدم Jury-matrix

و ليا شرط إضافية.

Construct Jury array

row n	Z^0	Z^1	Z^2	Z^{n-1}	Z^n
	a_0	a_1	a_2	a_{n-1}	a_n
1	a_0	a_1	a_2	a_{n-1}	a_n
2	a_n	a_{n-1}	a_{n-2}	a_1	a_0
3	b_0	b_1		b_{n-1}	
4	b_{n-1}	b_{n-2}		b_1	b_0
2n-3	γ_0	γ_1	γ_2	(ثلاثة عناصر آخر صف)	

لاحظ أن عدد
العناصر قد أصبح
أقل بمقدار
1

$$b_0 = \begin{vmatrix} a_0 & a_n \\ a_n & a_0 \end{vmatrix} \quad \text{أول عدد وآخر عمود}$$

$$b_1 = \begin{vmatrix} a_0 & a_{n-1} \\ a_n & a_1 \end{vmatrix} ; b_2 = \begin{vmatrix} a_0 & a_{n-2} \\ a_n & a_2 \end{vmatrix}$$

$$④ |b_0| > |b_{n-1}|$$

⋮

$$⑤ |r_0| > |r_{n-1}|$$

في الشرط الأول والثاني لو عرفت ب $z=1$ وطلع الناتج يساري 0 هتكمل باقي الشروط ولو اتحققت صيقتي (critically stable)

Ex ch. eqn $1 + \overline{GH}(z) = z^3 - 1.8z^2 + 1.05z - 0.2 = 0$

$$F(z) = \underbrace{z^3}_{a_3} - 1.8z^2 + 1.05z - \underbrace{0.2}_{a_0}$$

$$F(1) > 0 ??$$

$$F(1) = 0.05 > 0 \quad \checkmark$$

$$(-1)^n F(-1) > 0 ??$$

$$-1(-1 - 1.8 - 1.05 - 0.2) = 7.0 \quad \checkmark$$

$$③ |a_0| < |a_3| ?$$

$$0.2$$

1

$$|a_0| < |a_3| \quad \checkmark$$

97

	z^0	z^1	z^2	z^3
1	-0.2	1.05	-1.8	1
2	1	-1.8	1.05	-0.2
3	<u><u>b_0</u></u>	<u>b_1</u>	<u><u>b_2</u></u>	

3 عناصر
(آخر هيف)

$$b_0 = \begin{vmatrix} -0.2 & 1 \\ 1 & -0.2 \end{vmatrix} = -0.96$$

$$b_1 = \begin{vmatrix} -0.2 & -1.8 \\ 1 & 1.05 \end{vmatrix} = 1.59$$

$$b_2 = \begin{vmatrix} -0.2 & 1.05 \\ 1 & 1.8 \end{vmatrix} = -0.69$$

④ $|b_0| > |b_2|$?
 $0.96 > 0.69 \checkmark \checkmark$

→ system is stable.

$\overline{[10]}$

Ex 2 ch. equation

$$z^4 - 1.7z^3 + 1.04z^2 - 0.268z + 0.024 = 0$$

① $F(z=1) \geq 0$? ✓
 $\searrow \rightarrow 0.096$

② $(-1)^{n=4} F(-1) \geq 0$? ✓

③ $|a_0| < |a_4|$
 $0.024 < 1$ ✓

	z^0	z^1	z^2	z^3	z^4
1	0.024	-0.268	1.04	-1.7	1
2	1	-1.7	1.04	-0.268	0.024
3	b_0	b_1	b_2	b_3	
4	b_3	b_2	b_1	b_0	
5	c_0	c_1	c_2	← آخرها	

$$b_0 = \begin{vmatrix} 0.024 & 1 \\ 1 & 0.024 \end{vmatrix} \approx -0.999 \approx -1$$

$$b_1 = \begin{vmatrix} 0.024 & -1.7 \\ 1 & -0.268 \end{vmatrix} = 1.693$$

$$b_2 = \begin{vmatrix} 0.024 & 1.04 \\ 1 & 1.04 \end{vmatrix} = -1.015$$

$$b_3 = \begin{vmatrix} 0.024 & -0.268 \\ 1 & -1.7 \end{vmatrix} = 0.227$$

$$\textcircled{4} |b_0| = 1 > |b_3| = 0.227 \quad \checkmark$$

$$C_0 = \begin{vmatrix} -1 & 0.227 \\ 0.227 & -1 \end{vmatrix} = 0.943$$

$$C_2 = 0.63$$

$$\textcircled{5} |C_0| = 0.943 > |C_2| = 0.63 \quad \checkmark$$

→ System is stable.

EX:3 ch. eq

$$F(z) = z^3 + 3.3z^2 + 3z + 0.8 = 0$$

1) $F(z=1) = 0$? ✓

2) $(-1)^3 F(-1) > 0$?

↳ = -ve

(هذا الشرط لم يتحقق)

→ system unstable.

13